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Natural convection induced by the absorption of solar radiation: A review.

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Abstract

Natural convection primarily driven by the absorption of thermal energy known as penetrative or thermo-convection is a topic that generates attention due to its importance in various physical systems. A very common example of where this process can be found in geophysical systems such as lakes, where radiation induced natural convective transport have been seen to influence water temperature, biological activity and water quality.

The present paper reviews previous analytical, experimental and numerical studies reported in literature concerning natural convection driven by absorption of thermal radiation. Many of the reviewed studies were motivated by the interest of investigators to understand the physical processes in volumetric absorption thermal radiation in a fluid layer process and its associated energy transport.

In this class of problems, temperature fields are generally described as non-linear and the associated fluid flow is considered rather complex owing to coupling between the direct absorption of radiation and fluid flow. Parametric investigations for the effect of various parameters of interest such as the Rayleigh numbers, Prandtl numbers, spectral nature of incident radiative flux, optical depth, fluid absorptivity, aspect ratio, albedo and boundary emissivity on natural convection have been investigated.

The overall aim of the current review is to present a comprehensive review of the previous and recent approaches applied in the investigations of radiation induced natural convection in reservoirs. The paper also aims to contribute to improving the understanding of the physical processes, heat transfer and fluid dynamics associated with the thermal energy deposition into a fluid

layer. The paper is also highlight the potential application of this concept to help keep solar energy capture costs to a minimum and inform efficient designs of energy systems based on the concept of direct absorption of thermal energy inside a fluid layer.

Keywords: Natural convection, Radiation, Absorption, Experiment, Scaling, Simulation, analytical.

1. Introduction

Natural convection primarily driven force by the direct absorption of thermal energy also known as penetrative or thermo-convection is a subject of great interest to researchers. The energy transfer has been proven to have significant influence on fluid temperatures, biological activity, water quality and mixing of pollutants and sediments [1]. The most common example of this phenomenon is in physical systems such as the Earth's mantle, atmosphere, oceans and lakes [2, 3].

In recent years, this type of natural convection has attracted rapt attention in solar energy applications owing to the method of energy transfer which if efficiently exploited offers benefits such as prediction of water quality, and keeping solar energy capture costs to a minimum.

Critical to efficient thermo-fluid system designs based on the concept of volumetric absorption of solar radiation within a fluid layer is an accurate knowledge of the amount of solar energy absorbed by a fluid layer and different depths and an understanding the mechanisms of the associated physical processes [4].

Analytical [5], experimental [6], numerical [7–9] and scaling [9, 10] investigations on the direct absorption of thermal energy in a fluid layer and the induced thermal convective transport have been performed by different researchers under various contexts and boundary conditions.

Most investigations have been motivated by the quest to understand the process mechanisms and predict the temperature and fluid flow and their dependence on selected control parameters.

Absorption of solar radiation inside a fluid layer is selective and is dependent on the solar spectrum [11]. The absorption of solar radiation penetrating a fluid layer at different depths is generally explained by Beer Lambert law $I = I_0 e^{-\alpha y}$. The law describes the exponential decay of light penetrating a fluid layer at a rate dependent on the intensity, wavelength of light and the

30 attenuation coefficient of the fluid.

31 Theoretically, in direct absorption of solar radiation inside a fluid, longer
32 (red) radiation is absorbed within a few metres of penetration inside a fluid
33 layer and as such generates a stabilising effect and a stable stratification. The
34 shorter (blue) wavelengths mostly transmitted, if a fluid layer depth is shal-
35 low, the non absorbed solar radiation reaching the bottom surface absorbed
36 and re-emitted, develops a destabilising force which subsequently drives a
37 natural convection in that region.

38 While a considerable number of works concerning radiation induced natural
39 convection due to absorption of thermal energy has been conducted, at the
40 time of writing this paper, to the best of the authors knowledge, none of
41 the existing literature has conducted nor reported a comprehensive review of
42 subject of radiation-induced natural convection due to solar radiative heat-
43 ing. A number of review articles involving application of nanoparticles to
44 enhance solar energy absorption inside a working fluid have been published
45 over the past decade. Most of these publications have concerned with the
46 experimental and theoretical studies on optical properties and application
47 of nanofluids in direct absorption solar collectors [12]. The successful ap-
48 plication of nano-fluids in harvesting solar radiation by direct deposition is
49 dependent on the knowledge of their ability to absorb light energy of the
50 solar spectrum must be known. However, this topic is not within the scope
51 of the present paper.

52 The aim of this paper is to present a comprehensive review of investigations
53 performed on radiation induced natural convection in reservoirs to provide
54 a better understanding of the thermo-fluid transport process and the possi-
55 bility of utilising this concept to lower the cost of solar energy systems and
56 enhance system efficiency. Short comings and challenges associated with the
57 various investigation are also reported.

59 **2. Experimental studies for natural convection induced by volu-** 60 **metric absorption of solar radiation**

61 Penetrative convection in a cylindrical test cell was experimentally stud-
62 ied in Walden and Ahlers [13]. The experiments were concerned with the
63 measurements of the Rayleigh numbers and Nusselt numbers used to inves-
64 tigate penetrative convection in fluid layer. The departures of a non-OB
65 (Oberbeck and Boussinesq) system from a conventional OB approximation

66 as well as the nature and sizes of hysteresis loops were also investigated in this
 67 paper.
 68 Detailed description of the experimental apparatus and procedures are given
 69 elsewhere [14] and only briefly described here for brevity. The experiment
 70 was carried out in a Helium filled cylindrical test cell of height to diameter
 71 $(H/D) = 4.72$, quasi-statically heated at the bottom surface and the top
 72 (upper) surface maintained at a constant temperature. The top and bottom
 73 boundaries were made from copper, while side walls were made from stainless
 74 steel.
 75 Temperatures were set to a value T_0 which coincides with the maximum fluid
 76 density. The top fluid temperature was adjusted to attain a value about just
 77 greater than T_0 . The expansion coefficient which is kept positive and its value
 78 is then varied over a fluid layer height. The temperature dependence of the
 79 expansion coefficient was measured and then used to investigate the sources
 80 of departures from the OB approximation from a non OB approximation .
 81 A plot of the measured temperature taken at the mid plane of the test cell
 82 revealed a constant top temperature that monotonically increased with in-
 83 creased heating. At the lower surface the existence of penetrative convection
 84 was found to influence the development of a non-uniform temperature profile.
 85 A plot of the mean fluid density measured across a vertical profile demon-
 86 strated a convective instability evident by deep penetration of the convective
 87 flow into a locally stable layer for a temperature range between T_0 and T' ,
 88 where T_0 is the temperature at maximum fluid density and T' is an inter-
 89 mediate temperature for which a higher fluid density is attained than at the
 90 temperature at the bottom surface.
 91 The effect of control parameters; penetrative parameter (\mathcal{P}), Rayleigh (Ra)
 92 and Nusselt (Nu) number defined in terms of an effective thermal conductiv-
 93 ity on penetrative convective flow was investigated.
 94 An effective thermal conductivity was determined by imposing a time in-
 95 dependent heat current (q) and measuring the temperature increase of the
 96 bottom while holding constant the temperature of the top plate.
 97 The study identified time-dependent flow states which were generally non pe-
 98 riodic with characteristic mean frequencies. In a Boussinesq approximation
 99 case, a non-periodic time-dependent behaviour became evident close $Ra_c / 2$
 100 while for a non-Boussinesq approximation and penetrative convection a time-
 101 dependent flow is occurred much closer to R_c .
 102 When departure from buoyancy-driven convection was initiated, two dimen-
 103 sional roll patterns were found at a critical Rayleigh number Ra_c for a Boussi-

104 nesq approximation. For non-Boussinesq conditions over a finite range of
 105 Rayleigh numbers near the onset of convection the flow pattern consisted of
 106 hexagonal cells.
 107 Penetrative convection phenomena observed were explained based on three
 108 stages:

- 109 i. For $\mathcal{P} > 1$, the Nusselt number for the onset of time-dependent flow
 110 dropped sharply until the flow became time-dependent at the start of
 111 natural convection. For greater penetration a region of relative stability
 112 exists where the absence or presence of time dependence is influenced by
 113 the proximity of the system to a bifurcation between stable states [13];
- 114 ii. For $\mathcal{P} > 2$, a series of stable states closely spaced in Nusselt number was
 115 observed near Ra_c . Transition between states consists of three dimen-
 116 sional cells which are added or removed from the fluid volume to optimise
 117 convective transport at lateral boundaries. The spacing between identi-
 118 fied states decreased with increasing \mathcal{P} ;
- 119 iii. For $\mathcal{P} \leq 3$, the amplitude of hysteresis at Ra_c grew as \mathcal{P} increased toward
 120 2 and then declined at larger values of \mathcal{P} .

121 Webb and Viskanta [15] reported laboratory experiments for the radiation-
 122 induced natural convection occurring in a volumetrically heated test cell. The
 123 study aimed to describe the interaction of heat transfer and natural convec-
 124 tive fluid flow within a vertical fluid layer directly absorbing incident thermal
 125 energy incident from a horizontal direction.

126 Fig 1a shows a schematic of the experimental test apparatus and the inci-
 127 dent solar radiation. The experimental test cell with dimensions $48 \times 145 \times$
 128 41mm contained degassed water illuminated by a Quartz halogen lamp with
 129 parabolic dichroic (Philips 13117) mirrors through an optically transparent
 130 front vertical wall. The test cell external vertical side walls were insulated
 131 with 5cm Styrofoam insulation. The top of the test cell was left open for
 132 filling. The inner compartment was made from a black 12.7mm thick copper
 133 block wall. After about eight to ten hours when the system reached steady
 134 state, fluid temperatures were measured while florescent dye injection tech-
 135 nique was used to visualise fluid flow field.

136 Interferograms for a radiation flux incident on the water layer were recorded
 137 throughout experiments using a Mach-Zehnder interferometer. Interfero-
 138 grams recorded for incident flux corresponding to 1300W/m^2 for aspect ratios
 139 1 and 2 is presented in Fig 1b and Fig 1c for aspect ratio 1 and 2. The dif-
 140 ference in fringe densities seen at the respective vertical walls as shown in

Fig 1b and Fig 1c is indicative of absorption of radiation and induced natural induced convective heat transfer. Interpretation of the lines of constant intensity converted to isotherms found each fringe to approximately correspond to a temperature difference of 0.14°C . Incident radiation deposited correctly into fluid layers of high optical depth was completely attenuated before reaching the opaque walls.

Authors also used the interferograms to also analyse fluid flow patterns. Fluid flow induced by absorptive thermal radiation heating close to the transmitted surface rose to the top of the cavity. Fluid travelled along the top free surface and then into the a thin boundary layer that formed at the opaque wall. The flow observed from the interferograms was later confirmed by flow visualisation experiments. The flow structure found in this case is seen to be different from flows in cavities with differentially heated walls. Experimental results showed the central interior core to be stagnant and stratified. The flow structure lost a centrosymmetry characteristic of natural convection flows in cavities with differentially heated walls. The eddy centre was displaced to a position closer to the cooled side wall due to the direct heating of the core of the flow by solar radiation. [15].

Coates and Patterson [16] experimentally studied the unsteady natural convection in a rectangular tank directly absorbing thermal radiation. Experiments overall objective was to performed a series of laboratory experiments and used to test, verify and validate previous scaling and numerical experiments reported in previous studies.

In fig 2 is a schematic of the test cell used in the experiment. Experimental tank was a box made from Perspex with side walls 20mm thick and a 10mm thickness for walls elsewhere. The Perspex box was placed inside a constant temperature bath filled with filtered water to maintain the tank at constant temperature. The internal compartment consisted of three chambers of different sizes: a large tank of volume of 0.036m^3 , a small chamber of volume 0.0054m^3 and the last tank situated below the large and small tanks has a volume 0.018m^3 as shown in fig 2. The bottom tank's sole purpose was to control overheating at the lower boundary.

Perspex lids placed on the top of the tank was placed on spacers to formed a 4cm air gap and a lower lid placed directly on the water water surface to form a non slip boundary. Deionised water inside the test cell was directly irradiated by a 1000W quartz globe spot theatre lamp for maximum, intermediate and low intensities during the experiment [16].

Fluid temperatures and velocities were recorded using thermistors and pattern-

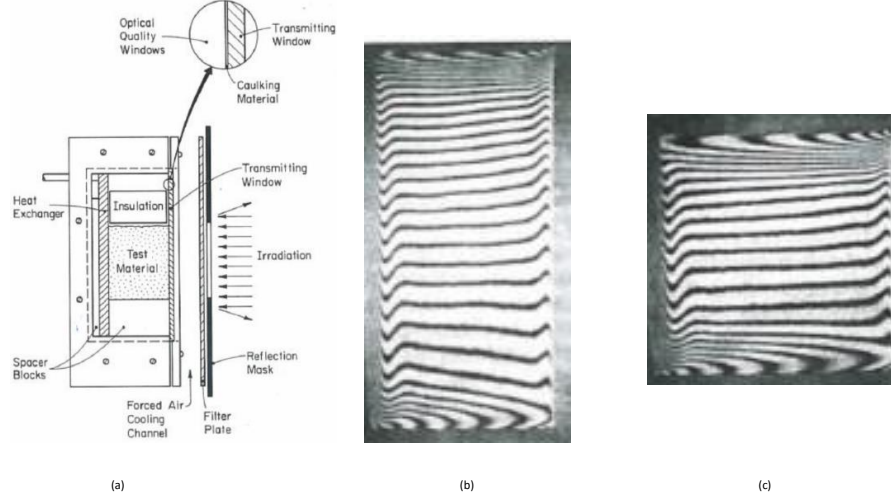


Figure 1: Experimental set up and results: (a) Test cell (b) Inferferograms for $H/W=2$ (Left) (c) Inferferograms for $H/W=1$ [15].

179 particles (Pliolite) tracer tracking technique.

180 Fig 3a shows a plot of experimental results for the measured temperature/depth
 181 data for three light intensities. Generally the results showed a decrease in fluid
 182 temperature with decreasing intensity. From the figure a non uniform tem-
 183 perature profile at all three intensities can easily be seen where temperatures
 184 are higher at lower fluid depths and dropped of to much lower values at great
 185 fluid depths .

186 In Fig 3b time history of the temperature difference recorded from the ther-
 187 mistor under maximum intensity at a fluid depth of 20mm and at distances
 188 20mm, 85mm, 150mm and 215mm from the light/dark boundary is pre-
 189 sented. Temperature increased approximately monotonically for each run at
 190 respective thermistors positions.

191 Flow intrusions were detected shortly after radiative heating was initiated
 192 that became distinct with further heating. Each time the intrusion reached
 193 the thermistor a sharp spike in the temperature was noticed. The sharp spike

194 in temperature is attributed to warmer water travelling faster and catching
 195 up with the cooler and slower water forming a sharp front.
 196 Identical trends were observed for the intermediate and low intensity experi-
 197 ments. Results are consistent with observations seen from the flow measure-
 198 ment. The main differences between results obtained at the three intensities
 199 are the magnitude of temperatures and possibly the timescale at which the
 200 intrusions occur.

201

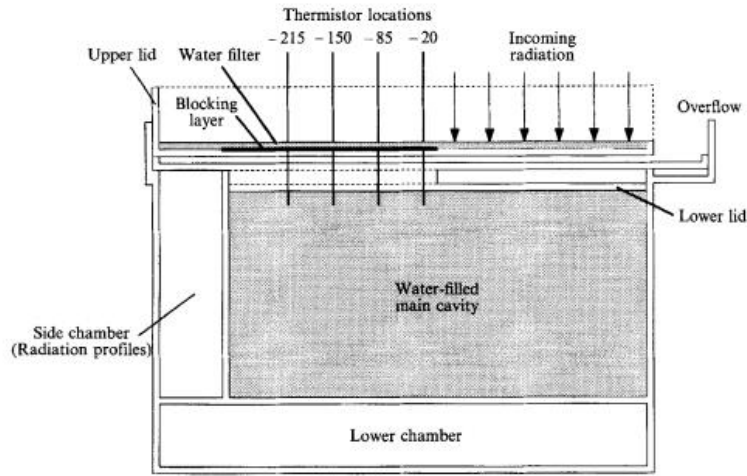


Figure 2: Schematic of the experimental setup showing the three compartments, two lids, thermistors and incoming radiation [16].

202 Natural convection induced by selective absorption of radiation was ex-
 203 perimentally studied in laboratory experiments performed by Krishnamurti
 204 [17]. This study aimed to demonstrate that convection can occur in a stably
 205 stratified fluid layers. In this experiment a DC voltage was applied across the
 206 top and bottom boundaries of a fluid layer containing dilute solution thymol
 207 blue (a pH indicator), turns the water colour orange, turns yellow if water
 208 pH is low and turns blue if the pH is high [17]. At the start of the experiment
 209 the fluid layer was subjected to steady and uniform radiation incident from
 210 above with light from a sodium vapour lamp. Fig 4 presents the experimen-

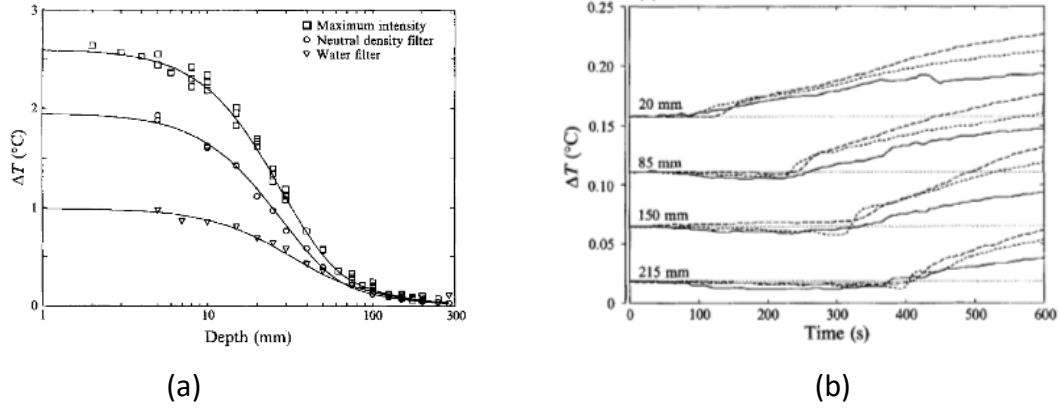


Figure 3: (a) A plot of the measured temperature/depth data for the three experiments, acquired after the water had been heated for one hour. (b) The time history of temperature increase from the thermistor data taken a depth of 20mm in the shaded region for three repeated runs. [42].

tal set up and apparatus used in Krishnamurti [17].
Visual observations showed that radiation travelled through the orange fluid and was negligibly absorbed. In the blue fluid radiation was seen to be strongly absorbed. Convective instability was generated from the heating of the blue fluid, evident by the blue fluid rising into the warm upper layer. Penetration of the blue fluid into the upper warm layers continued to occur as long as the fluid layer remained blue and the generated temperatures exceed the temperatures in the layer above [17]. Radiative heating was observed to only occur in the blue rising flow and not in the orange fluid.
In Figure 5 an image obtained at $Ra = 8.0 \times 10^5$ shows a strongly stable stratified layer with convective plumes cluster penetrate the lower fluid layer but confined below that upper stable stratification is shown. For a weakly stratified layer, plumes extend further and collect into a large cluster which exists as steady coherent structures.

Experimental investigations for the unsteady natural convection induced by absorption of radiation in a triangular tank is reported in Lei and Patter-

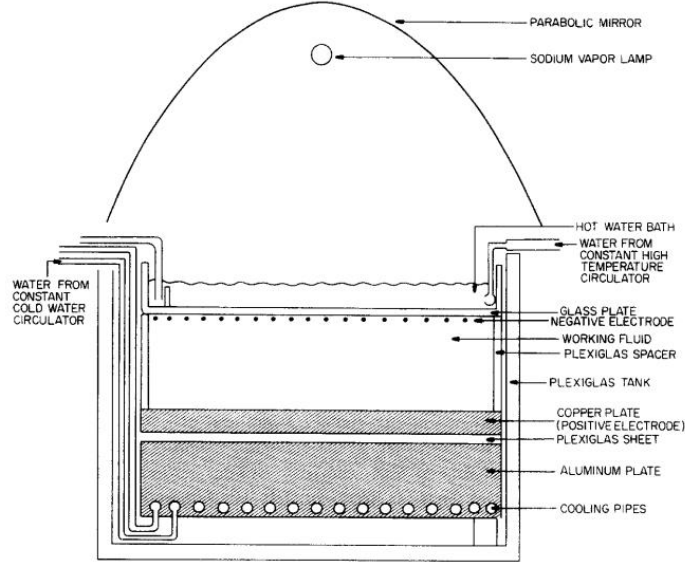


Fig. 1. Schematic diagram of the apparatus.

Figure 4: Schematic diagram of apparatus.

son [6, 7]. The aim of this experiment was to simulate the volumetric heating in a side arm in littoral waters due to solar radiative heating. For the experiment a shallow water filled triangular tank with dimensions $l=600$ mm and $h=60$ mm with a sloping and absorptive bottom was directly illuminated by a theatre spot light source at the top surface. Thermocouples situated at discrete locations along the sloping bottom measured fluid temperatures and fluid velocities were visualised using shadow technique. Fig 6 presents images of the water body at heating times 10s, 20s, 30s and 40s. In Fig 6a, $t=10$ s after the initiation of thermal radiation, the volumetric absorption of the incident radiation generated noticeable increase in water body temperature in downward direction. The non-absorbed radiation absorbed at the lower surface heats it and a thermal boundary layer is formed adjacent to the lower surface which continued to grow for maintained heating. At some stage when the thermal boundary layer became thermally unstable, thermal plumes are seen to grow out of the boundary as illustrated in Fig 6 b and c. In Fig 6d, further growth of the rising plumes within the lower fluid

245 column but confined below a stable thermal stratification is exhibited.
 246 Results demonstrate that thermal stratification of the water column and
 247 thermal plumes are important thermal features. The flow development is
 248 categorised by three distinct stages; (i) an initial stage marked by thermal
 249 boundary layer development (ii) a transitional marked by the existence of
 250 rising thermal plumes that extend out of the thermal boundary and into the
 251 fluid column and (iii) a quasi steady state stage. Authors suggested that
 252 their results obtained has not been previously been reported in literature.
 253 However, in spite of the significant findings reported the investigation, the
 254 experiment was limited to one set of the experiment test and was insufficient
 255 to draw a full picture of the flow. The set up did did not also permit a mean-
 256 ingful parametric investigation of this flow. The non linear temperature field
 257 and flow structures are consistent with previous studies.
 258 The characteristic zones in penetrative convective mixing in a stratified
 259 fluid layer were reported for laboratory experiments of Maroni and Canedese



Figure 5: Unstably stratified tank and a shallow convective cluster and plumes taken for $Ra = 8.0 \times 10^5$ [17].

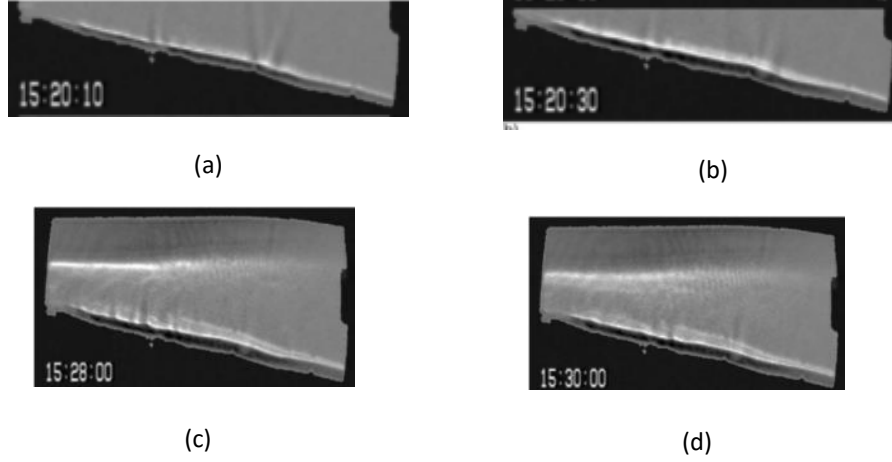


Figure 6: Shadowgraphs at (a) 10s (b) 20s (c) 30s (d) 40s [6]

[19].
 A 1000 W arc lamp directly heated water contained in a prismatic tank with a square base of dimensions $0.41 \times 0.41 \text{ m}^2$ and height of 0.40 m, insulated with Polystyrene sheets on all side walls. A beam stopper was used to control the depth of the illuminated area during the experiment.
 Twenty-seven thermocouples placed along a vertical line measured the vertical temperatures and a second set of thermocouples placed along a horizontal line on the lower boundary monitored the horizontal temperature profiles and horizontal homogeneity.
 Feature Tracking (FT) measured the fluid velocity along a vertical cross-section. Images of highly reflective tracers (pollen particles with average size of about $80 \mu\text{m}$) were recorded using two monochrome 8-bit CCD cameras with a time resolution of 25 frames per second (fps)[19]. Fig 7 a-c, presents images captured for shadow graphs recorded at experimental time, $t = 180\text{s}$, 400s and 919s . From these figures development of the thermal plumes with time is illustrated and the extent of extension from the lower surface into a region of stable stratification is easily seen. Visualisation performed also investigated the movement and shape of the interface between an stable and unstable layers.

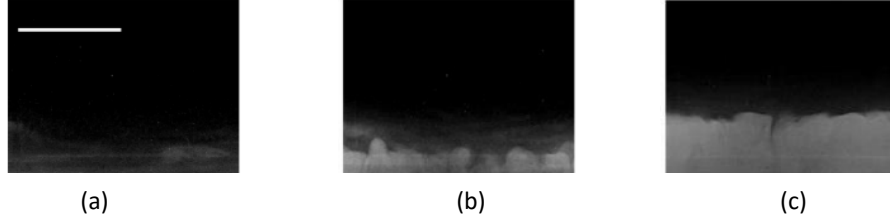


Figure 7: Visualisation of mixing layer evolution. Each snap shot corresponds to a progressive time (white line in frame is 10cm); (a) 180s, (b) 400s (c) 919s [19].

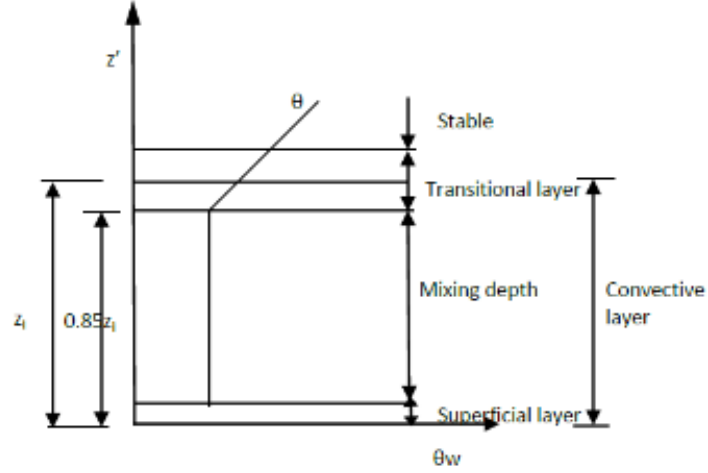


Figure 8: Schematic showing characteristic zones [19].

279 Characteristic zones for penetrative convection were identified in this
 280 study which shown in Fig 8:

- 281 i. Stable layer: is characterised by a stable temperature profile with con-
 282 stant gradient and zero convective heat flux .
- 283 ii. Mixing layer or transitional layer: Natural convection evident by the

284 existence of narrow plumes extend out of horizontal surfaces.
 285 iii. Boundary layer or mixing layer: Characterised with a typical thickness of
 286 about $0.85z_i$. The layer temperature and density profiles are constant.
 287 transitional layer, at around $0.85z_i < z < 1.2z_i$, the temperature profile
 288 increases and reaches a stable behaviour.

289 The flux passing through the interface between the mixing layer and the sta-
 290 ble layer was play a fundamental role in characterisation and forecasting of
 291 water quality in shallow water layers [19].

292 Dore et al. [20] advanced Maroni and Canedese [19] earlier experiments by
 293 using three-dimensional Particle Tracking Velocimetry (3D-PTV) technique
 294 to investigate penetrative convection in stratified fluids.

295 Boundary layer experiments captured the evolution of the mixing layer and
 296 its growth with time.

297 Fig 9a shows the results of the measured temperature time history. The pro-
 298 file as shown changes with time and three distinct characteristic regions : (a)
 299 a negative gradient in the boundary region associated with the existence of
 300 the thermal boundary layer, (b) a uniform temperature in the region where
 301 mixing occurs and (c) a region above the mixing layer, where the tempera-
 302 ture profile becomes a straight line of the initial stratification can be seen.
 303 Fig 9a presents the corresponding plot of the standard deviation of the verti-
 304 cal velocity component with time. The standard deviation very small at the
 305 beginning which later grew and covered a greater portion of the fluid depth
 306 as can be seen in Fig 9a. This finding supports previous results where the
 307 mixing layer associated with fluctuations in the measured velocity about a
 308 mean value. From the plot a sharp increase from the boundary with height
 309 can be seen.

310 A comparison of results showed that three-dimensional Particle Tracking Ve-
 311 locimetry (3D-PTV) technique is more accurate than a convectional scanning
 312 two-dimensional PTV for high tracer particle density. This is attributed to
 313 the fact that particles can be tracked directly in three-dimensional space
 314 rather than through matching of two-dimensional projections. Therefore, a
 315 three-dimensional PTV procedure would be more suitable for reconstructing
 316 a displacement field (i.e. particle trajectories) in both the mixing and the
 317 stable layer, allowing much more particles ($> 1,200$) to be tracked.

318
 319 Bednarz et al. [21] combined Particle Image Velocimetry (PIV) and Par-
 320 ticle Image Thermometry (PIT) techniques to experimentally investigate the

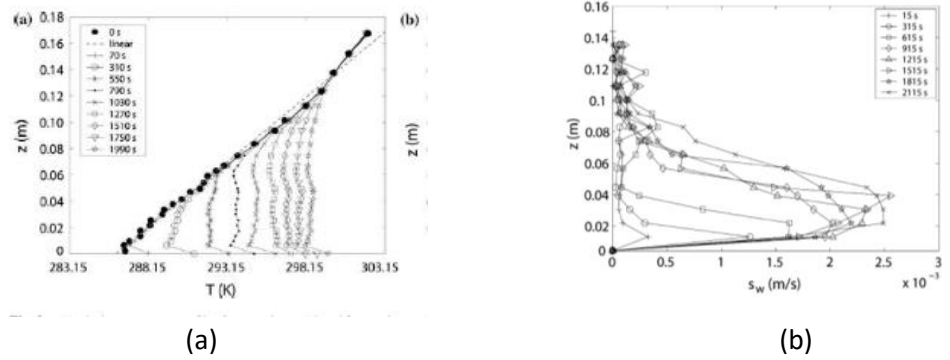


Figure 9: (a) Observed vertical temperature (b) Standard deviation profiles of the vertical velocity at different test times [20].

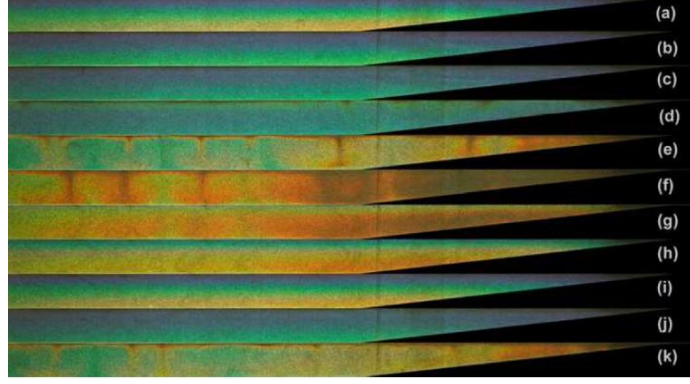
321 unsteady natural convection in a water filled reservoir model subjected to
 322 periodic thermal forcing. Experiments were conducted in a reservoir with
 323 dimensions $L=30\text{cm}$, $W=6\text{cm}$ and $H=1.5\text{cm}$; a wedge (inclination 0.1 , hor-
 324 izontal length $L_s=15\text{cm}$, $W=6\text{cm}$ and $H=1.5\text{cm}$) made from transparent
 325 Perspex. The central vertical cross-section of the flow domain was subjected
 326 to white light from a halogen lamp of a 150 W Liesegang 3000AF projector
 327 located about two meter from a mirror placed underneath the reservoir. Ex-
 328 ternal wall of the experimental reservoir were not insulated in order to allow
 329 illumination and visualisation of flow fields.

330 A quasi steady state natural convection within the trapezoidal reservoir
 331 whose temperature and flow fields are characterised by stable stratification
 332 of the water body during the heating phase and an unsteady mixing flow in
 333 the reservoir during the cooling phase [21].

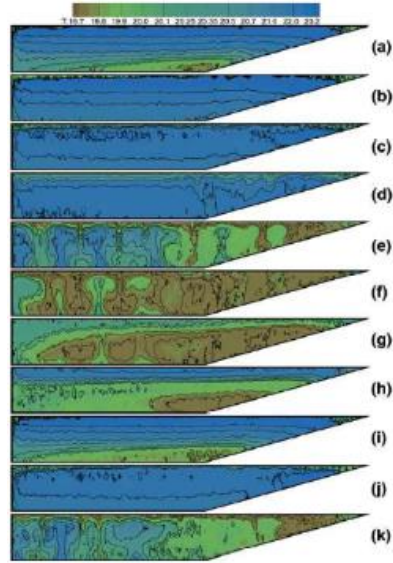
334 Photographic images were captured using a high resolution 112.8 Mega Pixel
 335 ($4368 \times 2912\text{ pixels}$) SLR digital camera (Canon EOS 5D).

336 Fig 10a shows images experimental images, extracted isotherms (Fig 10b)
 337 and velocity contours (Fig 10c) obtained from a periodic thermal forcing for
 338 $Gr=3.52 \times 10^4$ and $Pr=6.82$.

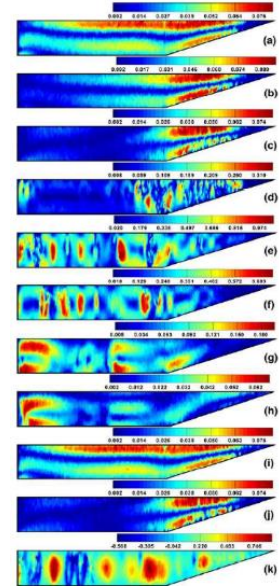
339 Natural convective circulation in the reservoir changed direction when ther-



(a)



(b)



(c)

Figure 10: Experimental results (a) Experimental images of the observed flow at $Pr=6.82$ and $Gr=3.52 \times 10^4$. (b) Isotherms extracted from the colour images in (a). (c) Velocity magnitude contours at different stages of the flow response to the periodic thermal forcing at $Pr=6.82$ and $Gr=3.52 \times 10^4$. [21]

mal forcing switched from cooling to heating and vice versa. Observation of the thermal instabilities developed within the reservoir, reveal important mechanisms for breaking up circulation and initiating a reverse flow circulation in the water body when a switch of the thermal forcing from heating to cooling occurred.

Calculations for the horizontal flow rate exchange obtained from the velocity measurements showed the overall strength of the circulation in the heating phase is significantly weaker than that obtained in the cooling phase. Results obtained for the flow development suggested the findings in this paper to be relevant to cloudy atmospheric conditions on days when solar radiation is not available (cloudy conditions). Under such conditions the flow is solely driven by ambient temperature changes and the cooling effects dominates the convective heat transfer within the reservoir.

Naghib et al. [22] more recently, conducted laboratory scale experiments to study natural convection boundary layer in water-filled square cavity under radiative heating from a halogen lamp. The study was motivated by the interest in understanding the mixing driven by the daytime direct absorption solar radiative heating occurring in the near shore regions of lakes established stability properties. The experimental apparatus and procedure fully described in Naghib et al. [22] and briefly discussed here. An square tank (140mm) assembled from double glazed glass side walls and black anodized aluminium bottom boundary was used. A 1000 W halogen lamp, with a colour temperature of 3000K and spectrum assumed to be similar to that for solar spectrum was used as the source of radiation.

The study simultaneously applied visualisation shadowgraphs and Particle Imaginary Velocimetry (PIV) techniques to visualise thermal plumes development due to induced instability. The authors highlighted the novelty in using concurrent PIV and shadowgraph technique for investigations for radiation induced natural convection of problem [22]. Fig 11 a-f presents the concurrent PIV/shadow graph images for a representative test run. These results can easily be matched with the different stages of flow development previously identified.

In Fig 11a and b shadowgraphs and PIV show the first stage associated with the flow development characterised by no fluid flow and thermal boundary layer development. In Fig 11c images shows thermal features, plume-like structures consistent with the second stage flow development marked by the onset of thermal instability. In Fig 11d growth of the thermal plumes into mushroom shape plumes that extend into the fluid column towards the top

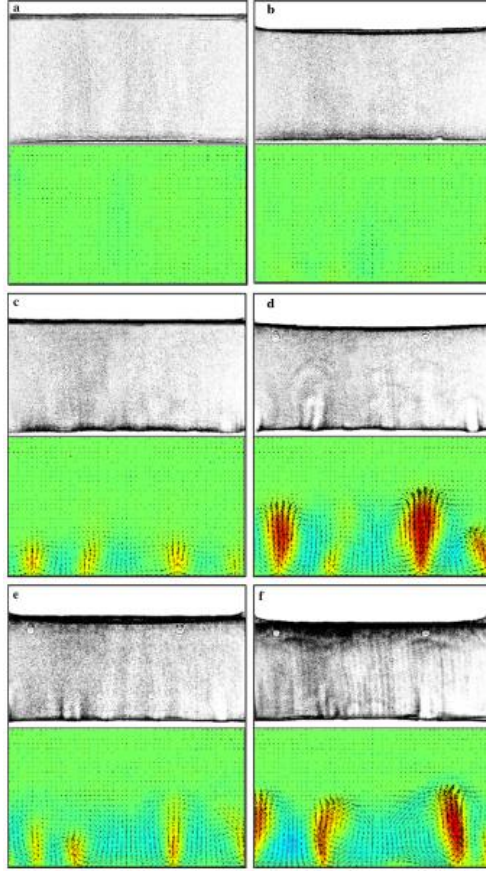


Figure 11: Concurrent PIV/shadowgraph images illustrating different flow stages. (a) 33 s. (b) 134 s. (c) 192 s. (d) 240 s. (e) 318 s. (f) 618 s.[22]

378 surface and impinge on the underside of the upper thermal boundary layer
 379 is shown. In the third flow development stage thermal plumes disappear
 380 and first and second stages are repeated in a cyclic pattern as illustrated in
 381 Fig 11e and f [22].

382 Thermal and flow features reported here are consistent with results of Lei
 383 and Patterson [6]. Direct comparisons between the experimental and scal-
 384 ing results were performed and were found to be in good agreement. Three
 385 dimensionality of the temperature and velocity field was also demonstrated

386 which have provided a basis for comparisons between two and three dimen-
387 sional flow behaviour.

388

389 Table 1 presents a summary of experimental work and experimental in-
390 vestigation for natural convection in enclosures driven by absorption of so-
391 lar radiation. Reviewed experimental results show a good consistency and
392 agreement amongst the many different. Therefore, results reported in exper-
393 iments are sufficient in providing good insight to the physical process and the
394 thermo-fluid transport. However a weakness for most of the studies is that
395 simulation of the complex reality of the instantaneous incoming radiation in-
396 tensity, is simplified by keeping a constant value from theatre and spot lights.
397 In nature, the intensity of the incident solar radiation is variable and is de-
398 pendent on the diurnal cycle, climate and cloud conditions [22]. The source
399 of radiation spectrum is considered to be a close approximation of the solar
400 spectrum, however has been shown not to be identical. As such, the plume
401 rise velocity could in the reality be of a higher order of magnitude, which
402 could consequently lead to significant higher rates of mixing beneath the top
403 thermally stratified layer than that investigated in the laboratory, owing to
404 the nature of the Rayleigh numbers 10^{11} - 10^{12} , which are higher than that
405 obtainable in the laboratory experiments [22].

406

407 3. Analytical (theoretical) analysis

408 Considerable number of studies on prediction models for radiation driven
409 natural convection using analytical techniques have been reported.
410 Early studies of Estoque [25] derived equations give in Eqn 1-3) for estimat-
411 ing the rate of heating due to penetrative convection and the subsequent
412 temperature of the environment due to near ground penetrative convection
413 on strong solar heating days. In Equation 1 prediction of the heating rate
414 strongly depends on a constant of proportionality (K) Tests on Equation 1
415 was performed using empirically derived data for the temperature difference
416 and the heat flux. Results showed that as the temperature difference di-
417 minished the heating stopped. This demonstrated a major limitation in its
418 predictions. Eqn 1 was modified in order to improve the applicability for
419 predictions. Eqn 2, the modified equation which included a term $M(Z)$. Test
420 using
421 Eqn 2 for the calculation of the theoretical local temperature using extrapo-

Table 1: Summary of the Experimental investigations on the radiation induced natural convection.

Author	Geometry	Method	Result and Remarks
Lei & Patterson [8]	Triangular	Shadowgraph visualisation technique.	Flow visualisation revealed an initial growth , transitional and quasi-steady stages of the flow development.
Walden & Ahlers [13]	Cylindrical	Thermocouples	Initial $N(R)$ for R near Ra_c was independent of P . For $P \lesssim 1$, two hysteresis loop formed, $P > 2$ multistability observed. For large P onset of time dependent flow occurred much closer to R_c than in boussinesq systems.
Webb & Viskanta [15]	Rectangular	Copper constantan thermocouples, Mach-Zehnder interferometer	Hydrodynamic boundary layers observed at vertical walls, being thinner at the cooled wall. A convection regime prevailed with thin boundary layers at the vertical walls and a stagnant, stably stratified central core. Difference in interference fringe densities at the vertical boundaries illustrated the absorption of radiation in the fluid.
Coates & Patterson [16]	Rectangular	light source used in the experiments was the Prolite company's Profile. Spot theatre lamp, containing a 1000 W quartz globe. Thermometric FP07 thermistors. velocities were measured by a pattern tracking technique	flow (inertial), becomes viscous or energy limited, dependent on the relation between Gr , η , and l_E , (through A_E). Convective regime flow regime is satisfied for $\eta_*^0 A_E^{-2} < Gr$ by the experiments. Validity of flow regime, and prediction of inertial and energy-limited velocity and time scales confirmed.
Krishnamurti [17]	Rectangular	A 35 W sodium vapour lamp consisting of a tube 1.3cm in diameter, 50 cm long, but doubled into a 25 cm long U-shaped tube, was placed along the focus of this parabolic cylindrical reflector. Heat flux meter	shallow convection plumes formed in a stably stratified fluid through the selective internal heating. Plumes in weakly stably stratified layers become organized into one large convective cluster.
Moroni & Cenedese [19]	Cylindrical	Thermocouples for temperature data and Laser Induced Fluorescence (LIF) and Feature Tracking (FT) image analysis techniques.	Pollutant dispersion phenomena are naturally described in the Lagrangian approach as the pollutant acts as a tag of the fluid particles. A matrix represents one of the possible tools available for quantifying particle dispersion during the evolution of the phenomenon.
Bednarz et.al [21]	Trapezoid	150W Liesegang 3000AF halogen projector lamp. PIV and PIT flow visualisation measurements. Thermochromatic liquid crystals	Stable stratification of water during heating phase and unsteady mixing flow during cooling phase. Thermal instabilities break up circulation and initiates reverse flow after thermal forcing switching
Naghib et al. [22]	Square	Theatre spotlight with a 1000-W halogen lamp. Radiometer (LI-COR LI-250). Shadowgraph and PIV flow visualisation measurements.	Onset of convection, plume rise height, and plume rise velocity determined from PIV and shadowgraph images. Agree with previously published scaling results.

lated surface temperature and heat flux data combined with observed temperature data were performed.

Results revealed that Eqns 1 and 2 are best used for dry convection. When convection is accompanied by condensation a general method suitable for

calculating the ambient, water vapour increase in the atmosphere due to penetrative convection, local increase in equivalent potential temperature for a conditionally unstable atmosphere and other related properties is required. Eqn 3 presents derived general equation [25] proposed. The strength of the method is dependent in the specification of $M(Z)$ in terms of large scale situations and the physical characteristics of the Earth surface.

432

$$\left(\frac{\delta\theta}{\delta T}\right)_c = K(\theta_0 - \theta) 0 \leq Z \leq Z_E \quad (1)$$

433

$$\left(\frac{\delta\theta}{\delta T}\right)_c = KM(Z)(\theta_0 - \theta) 0 \leq Z \leq Z_E \quad (2)$$

434

$$\left(\frac{\delta\theta}{\delta T}\right)_c = \frac{I_i M(\theta_0 - \theta)}{\int_{Z_{\delta Z}}^{Z_{iE}} \rho(\theta_0 - \theta) dz} \quad (3)$$

where subscript c, indicates temperature change due to convection and $K = I = I_0 / \int \rho(\theta_0 - \theta) dz$ is a proportionality constant, Z_i the height difference, Z_E is the equilibrium level. $M(Z) = 1 - Z/Z_E$ is arbitrary parameter proportional to mixing intensity.

439

Fig 12 presents results of the analytical computation plotted alongside a plot for a base case where $M(Z)=1$ against the height derivative of the observed heat flux. Satisfactory agreement can be seen between theoretical calculations and observed values for larger temperature differences at lower levels. The theory is seen to underestimate values at the lower end while overestimation is observed at upper level values. This is due to the fact that the proposed equations do not account for the effect of mixing between the thermal and the environment at lower levels.

448

Farrow and Patterson [5, 26] analytically investigated diurnal heating and cooling in a side-arm in a triangular domain with a small bottom slope. A two dimensional flow domain an infinite triangular wedge $0 < z < -Ax'$ heated thermal forcefully by a uniformly distributed radiative source term on an open top surface. x is the horizontal coordinate measured from the tip and z' is the vertical coordinate measure in the positive direction. The lower surface of the enclosure was considered to be heated by absorption and re-emittance of residual heat flux of a triangular cavity. The thermal forcing was modelled as an internal heating and cooling term

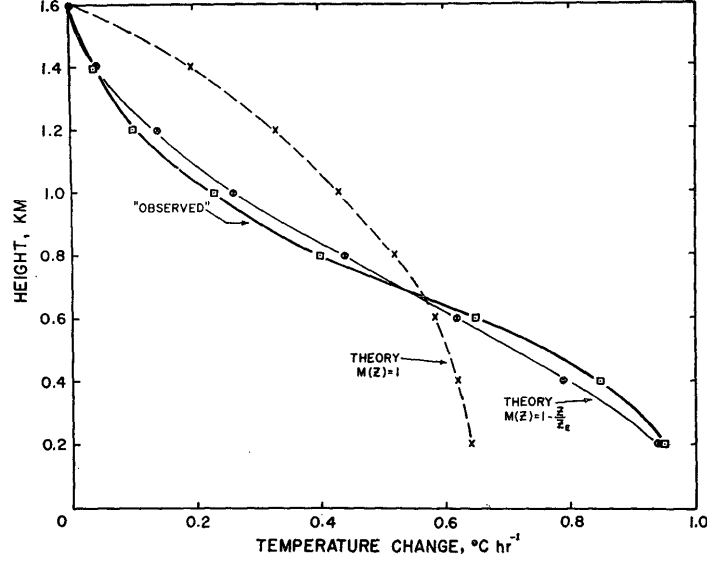


Figure 12: Comparison between theoretically derived values and observed temperature change [25].

458 integrated into a temperature equation of the form: $\frac{I_0}{\rho C_p A_x} \cos(2\pi/P)^\circ C s^{-1}$.
 459 The method used an expanded series in A^2 shown in equation 4 for a de-
 460 pendent variable T and A parameter which appears as even powers in the
 461 boundary value problems according to Cormack et al. [27]:

$$T = T^{(0)} + A^{(2)}T^{(2)} + A^{(4)}T^{(4)} + \dots \psi = \psi^{(0)} + A^{(2)}\psi^{(2)} + A^{(4)}\psi^{(4)} \quad (4)$$

462 For brevity details of the derivation are omitted here. The solution is
 463 obtained solving partial differential equations and taking Laplace transform.
 464 The solution for the horizontal velocity is given by:

$$U^{(0)}(x, z, t) = -\frac{1}{96\pi x^2} \sin(2\pi t)(z+x)(8z^2 + zx - x^2) \\ - 2x \sum_{n=1}^{\infty} \frac{1}{\beta_n^4 \sin \beta_n} \left((\beta_n \cos(\beta_n z/x) - \beta_n \cos \beta_n) \left(\frac{1}{2} + (\cos \beta_n - 1)/\beta_n^2 \right) \right)$$

$$\begin{aligned}
& -(\beta_n \cos(\beta_n z/x) - \sin \beta_n)(1 - \sin \beta_n/\beta_n) \\
& \times \left\{ \frac{(\beta_n/x)^2 \cos(2\pi t) + 2\pi \sin(2\pi t) - (\beta_n/x)^2 \exp - (\beta_n/x)^2 t}{(\beta_n/x)^4 + (2\pi)^2} \right\} \quad (5)
\end{aligned}$$

where β_n are the non-zero positive roots of the equation $\sin \beta_n = \cos \beta_n$.

Asymptotic solution obtained from analytical equations which were supported by numerical simulation showed more rapid heating and cooling to occur in the shallow regions which lead to distinct transition from the shallow regions. The presence of a sloping bottom resulted in the flux being greater at the shallow end than at the deeper end.

Farrow and Patterson [28] presented results for a more realistic daytime heating model driven by internal buoyancy sources and a boundary flux by accounting for depth dependent absorption of radiation. The problem considered in the context of the near-shore transport question suggested that the model can be relevant to any buoyancy-driven flow in which competing stability effects are present. The model combined the principle of both the attic model and an initial exposition of the geophysical model. Equation 6 presents Farrow and Patterson's [5] analytical solution. Asymptotic solutions to the equations were obtained and the results were compared to experimental results in a reservoir sidearm in which a significantly good agreement was obtained.

$$\begin{aligned}
T(y, t, h) = & \frac{t}{h} - \exp(y) + \frac{y^2}{2h} + y + \frac{1}{3}h + \frac{1}{h}(1 - \exp(-h)) \\
& - \frac{2}{h} \sum_{n=1}^{\infty} \left[\frac{1}{(n\pi/h)^2} - \frac{1 - (-1)^n \exp(-h)}{1 + (n\pi/h)^2} \right] \\
& \times \exp\left(-\left(\frac{n\pi}{h}\right)^2 t\right) \cos\left((n\pi/h)^2\right) \quad (6)
\end{aligned}$$

Hattori et al. [29], extended Farrow and Patterson [5] analytical solutions to determine the depth of the mixing layer and height of thermal plume travel in a parallelepiped shaped fluid layer with small slope situations and negligible horizontal convection. The equation which is derived from the one-dimensional, inhomogeneous heat equation. The solutions obtained

489 showed that the equation satisfied conditions at an initial stage temperature
 490 T_1 , but struggled to resolve unsteady boundary conditions introduced after
 491 the onset of instability. Vertical fluid temperature distributions (T_1) before
 492 and (T_2) after the onset of instability in a deep water layer with surface
 493 stratification is shown in Figure 13. Proceeding the onset of instability is an
 494 unsteady unstable flow characterised by bursting of buoyant plumes from the
 495 bottom boundary layer. Following the onset of thermal instability, the bot-
 496 tom boundary layer thickness becomes reduced due to a significant amount
 497 of heated fluid being lost in the form of rising plumes and limitation in the
 498 diffusive growth due to induced convection [29]. The occurrence and burst-
 499 ing of thermal plumes leads to homogeneity of fluid temperature over the
 500 plume height determined by the location of the point of neutral buoyancy.
 501 The surface stable layer remained stratified and is not destabilised by the
 502 boundary layer instability.

503 Fig 13 shows a schematic of the theoretical model plot defining a plume
 504 penetration height h_m . The plume rise height as shown by previous stud-
 505 ies is determined by the thermal stratification gradient [29]. From the plot
 506 the following was inferred: (1) The vertical temperature gradient over the
 507 lower mixed layer column significantly reduced as the temperature distribu-
 508 tion became homogenised due to plume activity. (2) The thickness of the
 509 lower boundary layer is insignificant compared to the total lower mixed layer
 510 thickness. (3) Temperature distribution over the lower mixed layer can be
 511 assumed uniform.

512 *3.1. Scaling analysis*

513 Scaling analysis has also been used by various researchers to study flow
 514 instability in radiation-induced natural convective flows with shallow wa-
 515 ter filled reservoirs, relevant to the bouyancy flows found in littoral regions.
 516 [9, 10, 30, 31].

517 Lei and Patterson [9, 18] established scales to quantify the flow properties
 518 for flow regimes and describe flow states for different parametric settings in
 519 a triangular reservoir subjected to solar radiative heating. The study consid-
 520 ered a two dimensional wedge of length (L) and maximum depth (H) whose
 521 top surface was directly heated by solar radiation. The sloping bottom sur-
 522 face was considered a black and capable of absorbing all the non absorbed
 523 radiation reaching it. Non slip conditions were applied at the bottom wall,
 524 end wall and at an open top surface.

525 From the scaling analysis possible flow stages of development dependent on

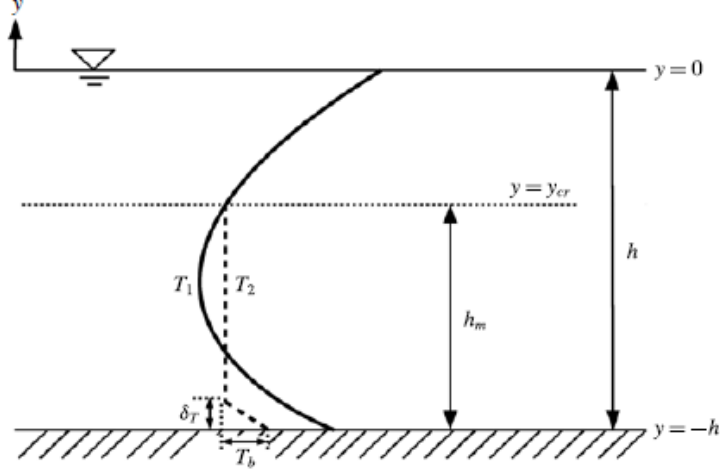


Figure 13: Schematic showing the bottom thermal boundary layer and the lower mixed layer thickness (h_m) with typical temperature distributions before (T_1) and after (T_2) the onset of instability in relatively deep waters [29].

the Rayleigh number were identified and described . The flow development stages were classified as conductive and convective/transitional and established scales were used to quantify their flow properties.

In an improved scaling analysis study conducted by Mao et al. [10], detailed scaling analysis for flow in a triangular reservoir with relatively large bottom slopes at high Rayleigh numbers were reported. The study aimed to reveal more detailed flows properties that was not captured by previous analysis. Properties of the thermal flow development were divided into three sub-domains, with the dominant mode of heat transfer changing from conduction into stable convection and subsequently into unstable convection as offshore distance increases. This scaling analysis demonstrated the extent of the unstable region and the onset time for instability at varying offshore distances. However, capability of the scaling analysis was limited in revealing further stability properties of the unstable region. With reference to a horizontal position, investigators derived critical function for the Rayleigh number from the scaling analysis used to identify the distinctness and stability of a thermal boundary layer. Four possible flow scenarios identified were found to be dependent on the bottom slope and the maximum water depth. For

544 the respective identified flow scenarios, the flow domain consisted of multiple
 545 sub-regions with unique characteristic thermal and flow features dependent
 546 on the Rayleigh number. The dividing points between neighbouring subre-
 547 gions were determined by direct comparisons of the critical functions of the
 548 Rayleigh number and a global Rayleigh number. Position-dependent scales
 549 were also established to estimate flow properties in different subregions. Au-
 550 thor's suggested that a power spectrum of the time series of flow properties
 551 would vary with offshore distance and would provide better insight into the
 552 variation of mixing and transport with offshore distance.
 553 The characteristics of instability in the transitional flow regime based on spec-
 554 tral analysis was reported by Mao et al. [10]. Harmonic frequency modes were
 555 observed for relatively low Rayleigh numbers. The power and prominence of
 556 instability at higher frequency modes increased with offshore distance and
 557 Rayleigh number. It was suggested that for a given offshore distance the
 558 frequency modes of instability are the same over the local depth, but the
 559 prominence of higher frequency modes increased with depth.

560
 561 Linear and non-linear analysis have also been applied to study radiation
 562 induced natural convection in reservoirs by different researches. Amongst
 563 these include works of Farrow and Patterson [28] who performed a linear
 564 stability analysis to study an asymptotic base flow with the bottom slope
 565 approaching zero, where a critical Rayleigh number for instability was de-
 566 rived as a function of the offshore distance. The analysis was however unable
 567 to characterize the instability properties in more developed flows.
 568 Hill [32], applied linear analysis and conditional nonlinear analysis based on
 569 the nonlinear energy theory to Krishnamurti's [17] study. The study defined
 570 a concentration based quadratic expression [32] that generated results con-
 571 sistent with a linear model internal heat source used by Krishnamurti [17],
 572 within defined parameter ranges. The linear theory was found to be only
 573 accurate enough to predict the onset of convective motion when the model
 574 for the internal heat source is predominantly linear. This is attributed to the
 575 presence of significantly large regions of potential sub-critical instabilities.
 576 Harfash [33, 34] extended the linear stability and the nonlinear energy sta-
 577 bility analysis of Hill [32] to three dimensional simulations.
 578 In this study the, onset of convection predicted in Krishnamurti's model [17]
 579 was explored to determine sub-critical regions where the linear instability
 580 and non-linear stability thresholds significantly deviated.
 581 Three dimensional governing equations were solved using an efficient, stable,

582 and accurate finite difference scheme using a velocityvorticity formulation
583 combined with a staggered grid. Implicit and explicit schemes used enforced
584 the free divergence equation used in the study.

585 Three different patterns dependent on the Rayleigh number (Ra) were ob-
586 tained for convection: For $Ra^2 < Ra_L$, the temperature, velocity, vorticity
587 and potential perturbations became negligible and the solution for a steady
588 state, before the linear thresholds are reached was obtained. The time re-
589 quired to reach the steady state increased as the Ra^2 increased. For R^2 is
590 close to Ra_L , solutions tend to a steady state which is different to the basic
591 steady state for $Ra^2 > Ra_L$ the solution do not attain any steady state and
592 oscillates. Ra_L is a thermal Rayleigh number

593 For three dimensional simulations, the inherent nature of the linear the-
594 ory only provided boundaries for instability, but did not predict anything
595 about instability, because of the presence of non-linear terms. For the non-
596 linear stability theory full assessments of any regions of subcritical instabil-
597 ities can be made. Therefore the non-linear stability theory is considered
598 more desirable than the linear stability theory when applied to predict the
599 stability analysis for the radiation induced convection in a three dimensional
600 domains.

601 Hattori [35] highlighted three common approaches are commonly used in
602 transient stability of penetrative convection induced by internal heating :
603 the amplification theory, the frozen time model, and the propagation theory
604 [35], for a stability analysis in transient systems [35–38].

605 The amplification theory requires specification of an initial arbitrary con-
606 dition for velocity and temperature a priori and directly solves a linearised
607 equation as an initial value problem. A major weakness of this model is its
608 need for physically reasonable initial conditions that must satisfy the speci-
609 fied boundary conditions.

610 In a frozen time model and propagation theory, linearised equations are re-
611 duced to a system of Ordinary Differential eigenvalue problems. Both meth-
612 ods do not depend on the arbitrariness due to the specification of initial
613 conditions inherent, as in the case of amplification theory.

614 The frozen time model and the propagation theory are used to investigate
615 the stability properties of the flow. Frozen time model and propagation the-
616 ory methods are computationally less expensive than an amplification theory
617 analysis [35].

618 The linear and oscillatory behaviours of the natural convection boundary
619 layer induced by the absorption of incident radiation in response to constant

620 and time-varying (ramp) thermal forcing using linear theory and direct sta-
621 bility analysis is reported in Hattori et al. [36]. Based on a quasi-static
622 linear stability analysis the fastest growth mode and its corresponding tem-
623 poral growth rate was determined. Findings are in excellent agreement with
624 results for the direct stability analysis previous numerical simulation. The
625 time and frequency scales of the unstable bottom thermal boundary layer
626 derived via a local Rayleigh number analysis revealed the boundary layer re-
627 sponse to the time-varying thermal forcing is in equilibrium with the forcing
628 intensity at each instant of time [35].

629 **4. Numerical studies for natural convection induced by volumetric** 630 **absorption of solar radiation**

631 Webb and Viskanta [39] performed two-dimensional simulations for the
632 natural convection in a geometry where the primary driving force for the nat-
633 ural convective motion is the volumetric absorption of thermal energy. Sim-
634 ulations were carried out in a two dimensional vertical semi transparent fluid
635 layer bounded by rigid solid walls on all sides. The top and bottom bound-
636 aries were assumed to be rigid and adiabatic. A constant isotopic incident
637 solar radiation was imposed at one of the vertical wall which was transparent
638 and transmitted the radiation. The incident radiation penetrated the fluid
639 layer and volumetrically heated it in the process and subsequently developed
640 a buoyancy driven flow. The opposite wall were considered to be opaque and
641 at a constant temperature.

642 Two-dimensional (non-dimensional) mass, momentum and energy equations
643 with a radiative divergence flux term that accounted for the depth-dependent
644 absorption of solar radiation based on a single weighted absorption coefficient
645 was solved by Finite Element Method. Governing equations were discretised
646 using the control volume scheme of Patankar while the SIMPLER algorithms
647 coupled the pressure and momentum [39]. The numerical model was val-
648 idated against previous experimental investigations of Webb and Viskanta
649 [39] and good agreement was obtained between results.

650 Numerical results were presented in the form of isotherms, streamline and
651 line plots. Fig 14 a-c shows isotherms, streamline and local convective heat
652 flux for a representative simulation at $Ra=10^7$. The heat transfer and fluid
653 flow in this study was observed to be different from those seen in differ-
654 ential heated cavity problems. A direct comparison between the numerical
655 results and interferograms obtained in previous experiments reveal identical

656 features.
657 The effect of defined parameters; modified Rayleigh number (Ra), Prandtl
658 number (Pr), fluid layer opacity, aspect ratio, wall reflectivity and heat loss
659 on natural convection were studied. At high Rayleigh numbers flow is seen
660 to completely lose centrosymmetry. As the the modified Rayleigh number
661 increased the existence of conduction, transition and boundary layer regimes
662 are evident. At low Pr number conduction is of greater importance while
663 convection dominated at high Pr numbers. Increasing fluid layer opacity
664 promoted the development of a boundary layer adjacent the transmitting
665 wall where very high thermal energy is deposited. Increased aspect ratio re-
666 sulted in a reduction in a stagnant central core in the boundary layer regime.
667 Decrease in the convective heat loss and increase in opaque wall reflectivity
668 slightly promoted convective flow.
669

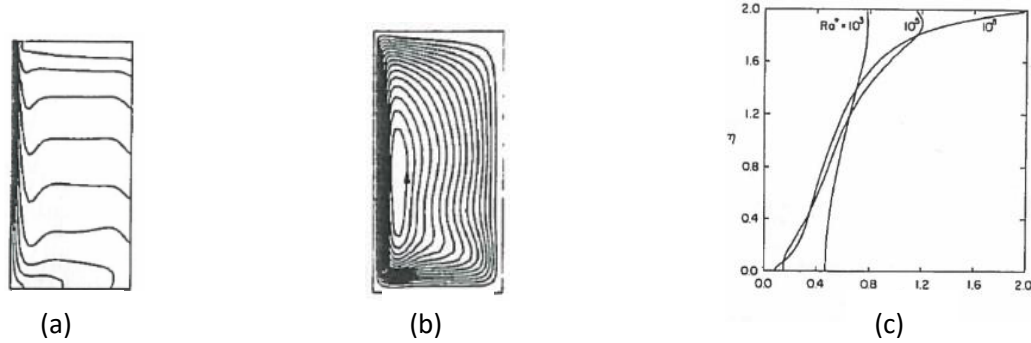


Figure 14: (a) Isotherms for temperature distribution (b) Streamlines for slow structure at for $Ra=10^7$ (c) Local convective heat transfer flux at the isothermal wall for $Ra=10^3$, 10^5 and 10^8 [15].

670 Onyegegbu [40] performed two-dimensional simulations for the solar ra-
671 diation natural convection in stagnant water layers. The study considered a
672 water filled rectangular tank of depth (H) and width ($2W$), insulated on ver-
673 tical side walls and bottom boundary and subjected to a time variant solar
674 radiation at the open top surface. Convection was assumed to be two dimen-
675 sional and flow is considered to be symmetric about the centre plane. The

676 model solved two-dimensional governing partial differential equations using
677 Finite Difference Methods. An unconditionally stable alternating direction
678 implicit method was applied to solve parabolic equations. The Non-linear
679 convective terms were represented using second up-wind differencing meth-
680 ods. The radiative heat source was obtained from solving the Radiative
681 Transfer Equations using a P-1 approximation.

682 Results obtained showed the induced natural convection set in the form of
683 two counter-rotating eddies of unequal strength which has significant influ-
684 ence the temperature field of the fluid layer. This phenomenon occurred for
685 Grashof numbers as low as 10^3 . Effects of parameters; $10^3 \leq Ra \leq 10^8$; 0.01
686 $\leq Pr \leq 1000$, $1 \leq A \leq 5$ and $0 \leq Bi \leq 5.0$ on temperature distribution,
687 local heat transfer and flow structures was performed. Results revealed that
688 convection is promoted by low fluid optical thickness and increased albedo.
689 The effect of the bottom boundary emissivity was significant only at low and
690 intermediate optical thickness where low boundary emissivity suppressed con-
691 vection.

692 Flow pattern characterised in three different regimes as intermittent con-
693 vection, steady state convection and free-convection were reported in the a
694 numerical study of Verevochkin and Startsev [41]. The result was reported
695 for the thermal convection in a horizontal water layer cooled from the top and
696 absorbing solar radiation [41]. The flow transition occurred for different com-
697 bination of the ratio of the downward solar radiation flux at a depth (J_0) to
698 heat flux through the interface (Q), and at almost the same Rayleigh number.

699
700 Two and three dimensional numerical investigations of unsteady natural
701 convection induced by absorption of radiation were performed for shallow
702 water filled triangular enclosures with sloping bottoms and subjected to solar
703 radiative heating [7–9]. The study was carried out to understand buoyancy
704 driven flows with respect to the daytime natural convection in a side arm
705 and in littoral regions [6–9].

706 Two and three dimensional momentum, energy and conservation equation
707 Governing equations were solved using a Finite Difference scheme. A stan-
708 dard Second Order Central differencing scheme is used for spatial derivative.
709 Non linear terms in the governing equation were solved using the second or-
710 der upwind scheme. Details of the numerical schemes can be found in [8, 9].
711 Fig 15 shows results from the two dimensional simulations at times corre-
712 sponding to 25s, 328s, 5302s and 5049s respectively. Images reveal features
713 consistent and can easily be matched with the early, transitional and a quasi-

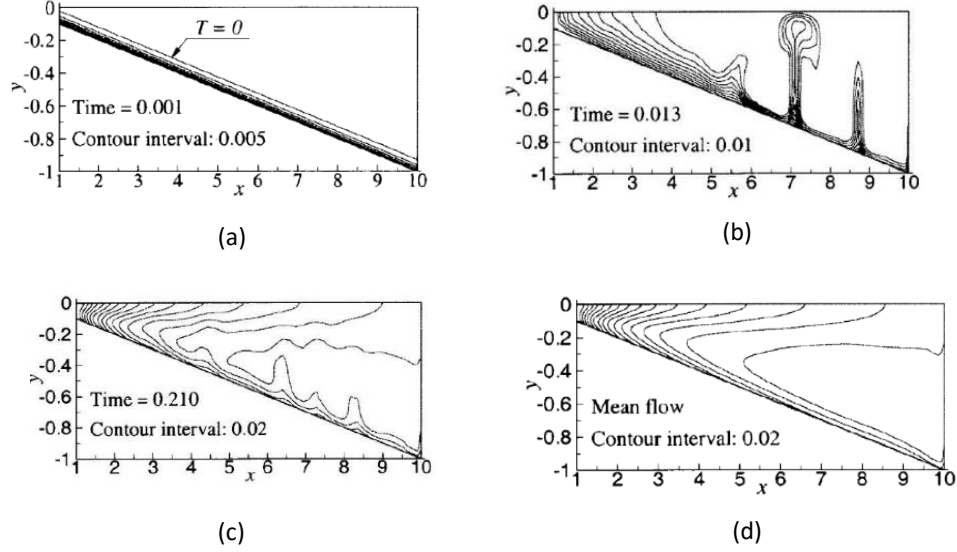


Figure 15: Two dimensional temperature contours during various heating times (a) $t=0.001$ ($t=25$) (b) $t=0.013$ (328s) (c) $t=0.21$ (5302 s) (d) $t = 0.200$ (5049s) [7].

714 steady flow stages. Good agreement between the two-dimensional simulation
 715 and observations from a flow visualisation experiment was obtained.
 716 Fig 16a-c presents images for the corresponding three dimensional simulated
 717 iso-surfaces for radiation induced natural convection in a triangular enclo-
 718 sure for various time steps. Iso-surfaces can be matched with the three
 719 stages of the flow development: an initial stage, a transitional stage and a
 720 quasi-steady stage. The heat transfer at the initial stage was dominated by
 721 conduction from the sloping bottom. The transitional stage started with the
 722 onset of instabilities, and the quasi-steady state was characterised by steady
 723 growth of a spatially averaged temperature. The results were also found
 724 to be consistent with observations from a flow visualisation experiment and
 725 the two-dimensional simulations. Fig 16d, presents a plot for the two and
 726 three dimensional time histories for the integrated horizontal heat transfer
 727 rate from two dimensional and three dimensional simulations. While the
 728 previously identified three stages of flow development can be easily seen for

729 these plots, discrepancies in time time-scales and the duration of occurrence
730 exist. This discrepancies between plots is attributed to non inclusion of the
731 azimuthal orientation in the two dimensional simulation.
732

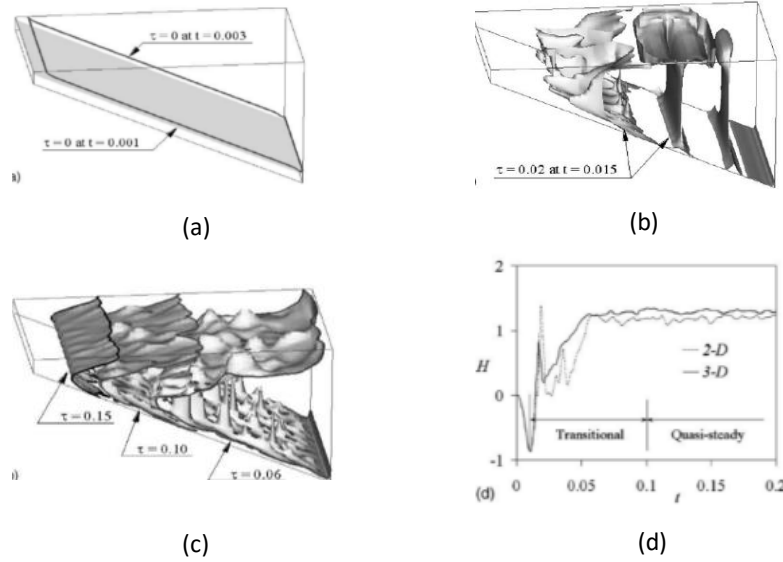


Figure 16: Three dimensional temperature iso-surfaces during various heating times (a) $t=0.001$ ($t=25s$) (b) $t=0.015$ ($t=379s$) (c) $t=0.21$ ($t= 5302s$) (d) Time series of integrated horizontal heat transfer rate [8].

733 In recent studies, three-dimensional simulations and scaling analysis for
734 the natural convection in reservoirs filled with water and subjected to solar
735 radiation was reported by Hattori et al. [29, 35]. The geometry studied is
736 considered to be relevant to deep water bodies subjected to top solar radiative heating [29].
737
738 Direct Numerical Simulations (DNS) were applied to solve the coupled three
739 dimensional mass, momentum and energy equations. An internal heat source
740 term in the governing equations is defined as $s(y) = \frac{I_0}{\rho_0 C_p} \eta e^{\eta y} - I_0 \rho_0 C_p h$, where

741 I_0 is the solar intensity at water surface, C_p is specific heat capacity, η is the
742 bulk attenuation coefficient.

743 The governing equations was solved using a non-staggered, Cartesian mesh,
744 Finite Volume code (SNS code), developed in Fortran90 language. The SNS
745 code is generally based on a fractional step method, with Adams Bashforth
746 and Crank-Nicolson time discretisation schemes used for the advection and
747 diffusion terms. Second-order central differencing schemes are used for the
748 spatial discretisations of diffusion and advection terms, while a Bi-Conjugate
749 Gradient Stabilised method (BICGSTAB) with a SIP preconditioner is used
750 for Poisson pressure correction.

751 Results described a non-linear temperature distribution consisting of two
752 distinct adjacent layers; an upper stratification region due direct absorp-
753 tion of radiation, and an unstable boundary layer due to the absorption and
754 re-emission of the residual radiation by the bottom surface [29]. The exis-
755 tence of a non-linear temperature stratification is seen to physically limit the
756 thermal plumes maximum height and mixing depth. In other words the non-
757 linear temperature stratification was considered to determine the penetration
758 length scale of the plumes and the lower mixed layer thickness. On the basis
759 of theoretical calculations the lower mixed layer thickness is suggested to be
760 equal to the attenuation length of the radiation [29].

761 Fig 17 presents temperature and flow contours for $Ra = 1.27 \times 10^9$, $h\eta =$
762 0.72 and dimensionless time, $t = 0.0026 - 0.0039$. In Fig 17a, tempera-
763 ture iso-surfaces show a thermal boundary laying approximately parallel to
764 the bottom boundary. In Fig 17b, thermal instability is introduced within
765 the boundary layer illustrated by evolution of thermal plumes. In Fig 17c,
766 thermal plume emerge from the thermal boundary, while in Fig 17d thermal
767 plume fully develop and extend into the fluid column. In Fig 17e and Fig 17f
768 show the Iso-surface of vorticities for the x and z components at position x
769 $= z = 19.4$. Dark and light patterns depict positive and negative vorticities.
770 Iso-surfaces of vorticities also indicate the formation of convective rolls with a
771 distinct dominant wavelength in each direction. These results are supported
772 by experimental and scaling analysis results reported in previous sections.

773
774 Turbulent natural convection simulations for the daytime heating and
775 night time cooling in a shallow tetrahedron domain was reported in Dittko
776 et al. [43]. The tetrahedron domain studied was considered to be a more real-
777 istic representation of three-dimensional approximation of a lake or reservoir
778 sidearm compared to a previously used two [7] and three-dimensional tri-

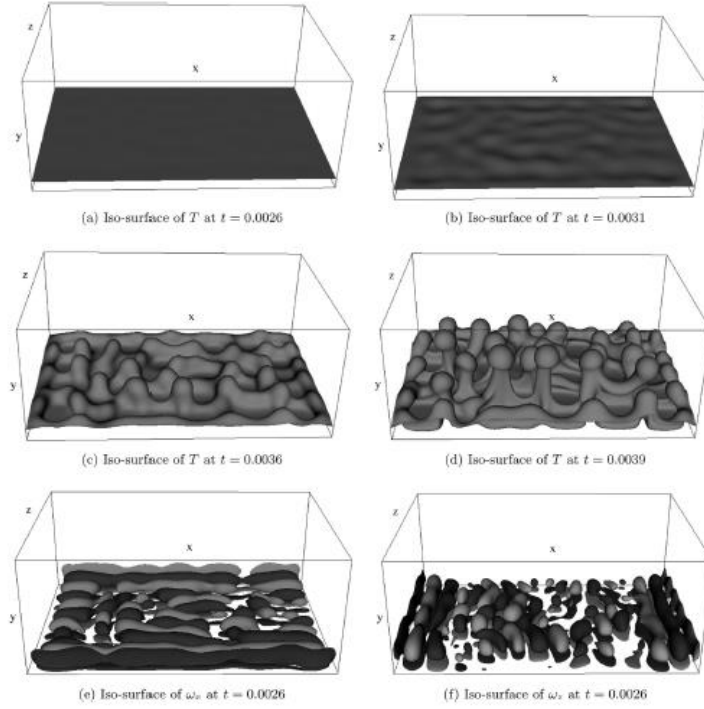


Figure 17: Flow structures for a case with $Ra=1.27 \times 10^9$ [29].

angular domain reported previously. The two dimensional reservoir model studied consisted of a region with a sloping bottom and one with a uniform water depth. The model has a total length L , maximum water depth of h and a slope inclination of $A = 0.1$.

Governing equations were solved by finite volume, fractional step, pressure correction method in a PUFFIN (Particles IN Unsteady Fluid Flow) solver. Fourth Order central difference scheme with the ULTRA flux limiter was used for the advection scheme. Time stepping was based on a second order hybrid Adams-Bashforth/Crank-Nicolson scheme. A staggered Cartesian grid is used along with the localised dynamic Smagorinsky model to account for subgrid scale turbulence effects [43].

Fig 18a shows temperature contours in a tetrahedron under solar radiation with $Ra=35,000$ while shows Fig 18b velocity contours in the x-direction in a tetrahedron subject to cooling with $Ra=1.6 \times 10^5$.

793 A thermal boundary layer formed just above the bottom surface is seen from
794 this plot. In Fig 18a conduction heat transfer is dominant close to the do-
795 main tip which is demonstrated by the vertical contours in the figure. Curved
796 contours occurring away from the shallow tip illustrate the presence of a con-
797 vection dominated flow. The flow seen maintained the expected general trend
798 to travel up along the sloped bottom boundary and along the surface. The
799 result supports the assumptions made in the scaling analysis for the two di-
800 mensional flow in the x-direction.

801 Fig 18b the contours shown indicate that on unstable flow occurs. A two
802 return flow is exhibited with the large area of the domain and a flow down
803 the bottom slope occurs over the entire length of the domain. The flow in-
804 dicates a strong volumetric transfer rate to and from the shallow tip down
805 the bottom edge of the domain. Effects of end wall as also shown to have a
806 strong effect of flow.

807 Calculations for the horizontal heat transfer and volumetric flow rates re-
808 vealed significant changes to the flow and introduced some complex features
809 such as helical flow towards and away from the tip.

810

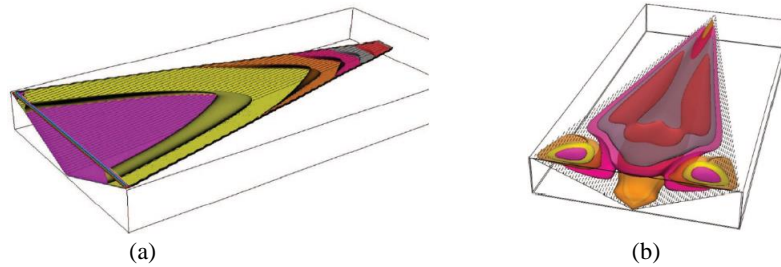


Figure 18: (a) Normalised mean temperature contours at intervals of 7.5 in a tetrahedron under solar radiation with $Ra=35000$ (b) Contours for the velocity in the x-direction in a tetrahedron subject to cooling with $Ra=1.6 \times 10^5$ [43].

811 Numerical simulations concerned with the transient flow response in a
812 reservoir model subjected to periodic heating and cooling (thermal forcing)
813 at the water surface was reported by Bednarz et al. [44–46]. The study
814 was concerned with understanding of the flow mechanisms and prediction

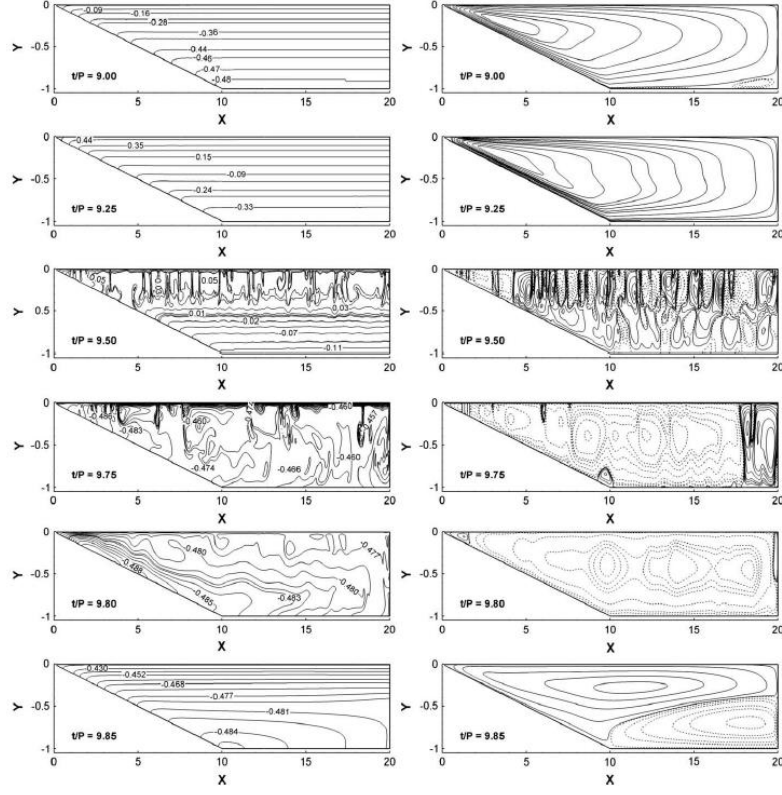


Figure 19: Different stages of the flow response to the diurnal thermal forcing in shallow waters of a reservoir at $Gr=10^7$. Left: Isotherms, right: Streamlines [44].

815 of the transport of nutrients and pollutants across reservoirs. The overall
816 objective of this study was to advance previous numerical investigation for
817 a reservoir model with thermal forcing imposed at the top surface. Periodic
818 thermal forcing considered for this simulation is obtained by varying surface
819 temperature which estimated the effect of varying ambient temperature with
820 no radiation input as could be experienced during cloudy atmospheric con-
821 ditions.

822 Dimensionalized governing equations were solved numerically with Finite
823 Volume method with a double precision solver. For all calculations a second-
824 order time formulation was employed. Spatial derivatives were solved using
825 a third-order MUSCL scheme obtained by combining a central differencing
826 scheme and a second-order upwind scheme. The SIMPLE algorithm is used
827 for the Pressure velocity coupling.

828 Fig 19 presents the isotherms (left column) and corresponding streamlines
829 (right column) at representative times over a typical diurnal cycle for Grashof
830 numbers $(Gr)=10^7$.

831 Stable stratification of the water body during the heating phase and un-
832 steady mixing flow in the reservoir during the cooling phase was obtained.
833 The unsteady flow structures and stable stratification obtained in the numer-
834 ical simulations are identical to those observed in experiments [21] carried
835 out in a reservoir model cooled and heated from above in diurnal cycles. The
836 development of unsteady convection described was based on experimental
837 flow visualisation and quantitative temperature and velocity measurements
838 using thermo-chromic liquid crystals. Developed thermal instabilities de-
839 stroyed the residual circulation and initiated a reverse flow circulation in
840 deep water when the thermal conditions changed from heating to cooling.
841 For sufficiently strong cooling, a clear undercurrent was formed which drove
842 cold water to the deep region of the reservoir. Heating from the water sur-
843 face generated a stable large-scale convective roll. Results also showed that
844 the primary convective circulation in the reservoir changes its direction (flow
845 reversal) when the thermal forcing changes from cooling to heating and vice
846 versa. The time lag of the flow response to the change of the thermal forcing
847 from cooling to heating depended on the strength of thermal forcing charac-
848 terised by the Grashof number.

849 The calculated horizontal exchange flow rates from the velocity fields showed
850 the overall strength of the circulation in the heating phase is significantly
851 weaker than in the cooling phase. Thus suggesting that during cloudy con-
852 ditions, the cooling effect dominates the convective motion in reservoirs.

853 The numerical results also presented indicated that the convective motions
854 in reservoirs in diurnal cycles are very important from environmental and
855 ecological points of view. Water circulation driven by horizontal thermal
856 gradients may cause the transport of small, suspended pollutants, biological
857 particles or dissolved constituents into or from deep water regions and thus
858 plays a significant role in determining water quality. The paper however,
859 considered only numerical results of convective motion in diurnal cycles and

recommended further study of particle transport to help incorporate relative phase and motions of particles into calculation and applied to the ecological transport models.

Amber and O'Donovan [47] presented numerical simulations for the natural convection induced by direct absorption of concentrated solar radiation in molten salt filled enclosures for height to diameter ratios (H/D) of 0.5, 1 and 2 and Rayleigh numbers $10^7 - 10^{11}$. The domain studied was made up of fluid cavity bounded by rigid adiabatic vertical walls, a heat-conducting bottom wall of finite thickness and an open adiabatic top surface, directly irradiated by a non- uniform concentrated solar flux. The simulation was conducted for the particular case that a salt volume is first heated by direct absorption of solar radiation and then by contributions from the lower boundary heated by the absorption of the radiation transmitted to it.

Time dependent two dimensional Navier Stokes Governing equations that included a depth dependent volumetric heat source was solved by Finite Element Method. Second-order and linear elements discretisation were applied for the velocity and the pressure field, (P_2+P_1). An unstructured mesh consisting of triangular mesh elements is adopted in the simulation to account for the variability of the formed thermal plumes and its occurrence at arbitrary locations [47]. The study also considered the temperature dependence of the thermophysical properties of working fluid. A nonlinear temperature profile consisting of a hot stable stratified fluid above a cooler convecting layers where thermocapilarity and buoyancy effects are evident was observed. Fluid flow development in the lower layer is found to exhibits three stages. Natural convection was found to decrease with increasing aspect ratio and increased with increasing Rayleigh number. The Nusselt-Rayleigh number relationship was found to not be linear.

Fig 20 shows the time averaged temperature contours, averaged over in the transitional stage for values: (a) $Ra = 10^7$, (b) $Ra = 10^8$, (c) $Ra = 10^9$, (d) $Ra = 10^{10}$, (e) $Ra = 10^{11}$ at $H/D = 0.5, 1$ and 2 . At $H/D = 0.5$, plume penetration height and the intensity of plume occurrences increased with Ra . The flow in the lower layer exhibited an increasingly complex behaviour with increased Ra . Observations for $H/D = 1$ and 2 reveal that for each aspect ratio increased flows are found to occur with increasing Ra . From these plots it can also be seen that fluid flow corresponding to a particular Ra decreases with increasing aspect ratio.

Amber and O'Donovan [47] investigated the effect of inclination angle ($0 < \phi < 60^\circ$) natural convection in a molten salt-filled enclosure induced

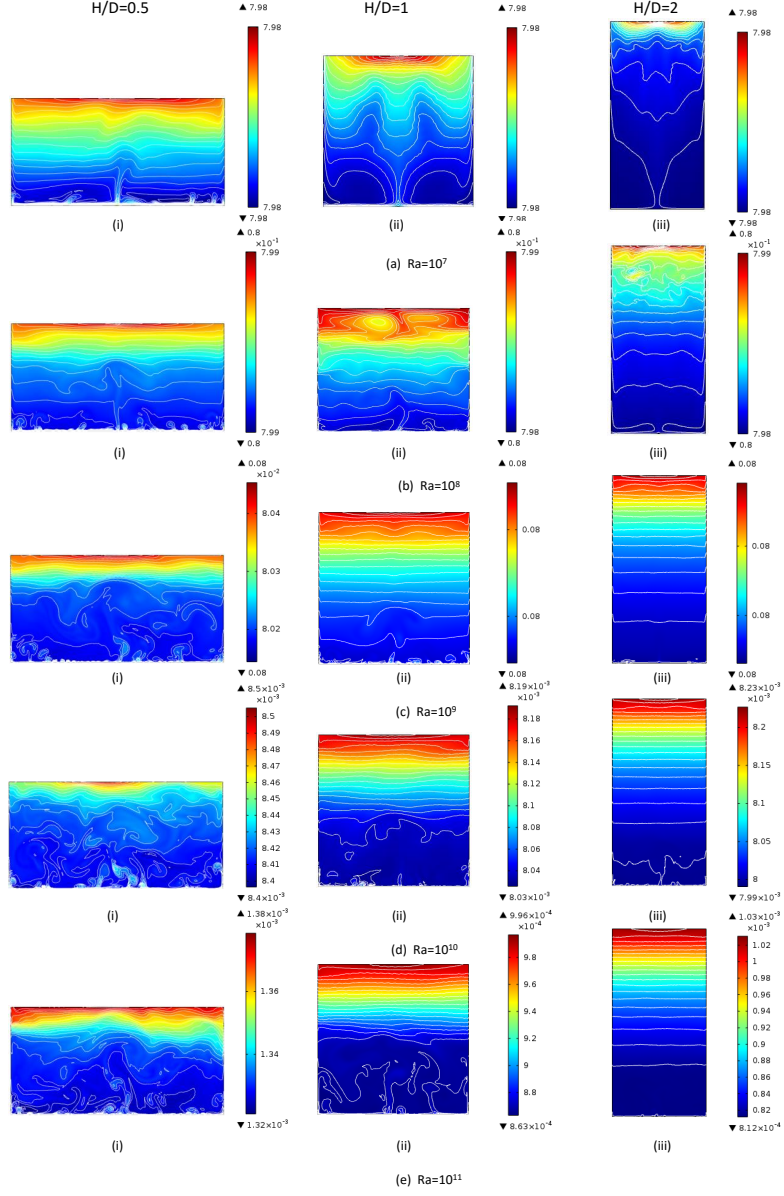


Figure 20: Time averaged temperature contours excluding the quasi steady regime at $H/D=0.5$ for Rayleigh numbers a) $Ra=10^7$ b) $Ra=10^8$ c) $Ra=10^9$ d) $Ra=10^{10}$ e) $Ra=10^{11}$ for $H/D=0.5$ (left column), $H/D=1$ (middle column) and $H/D=2$ (right column) [47].

by direct absorption of non-uniform concentrated solar radiation. A two-dimensional inclined enclosure with $H/D = 1$, whose vertical walls were inclined at an angle to the gravity vector was considered. All boundary walls were assumed to be rigid and adiabatic except for the top boundary which is an open stress-free boundary. Bottom boundary is of finite thickness (dx), and absorb all radiation reaching it. The lower boundary then heats the plate and drive natural convection. The working fluid (KNO_3 - $NaNO_3$ salt), within the domain walls is initially at rest and temperature T_0 . At time $t = 0$, a non-uniform concentrated radiation flux is imposed and thereafter maintained.

The inclined geometry is identical to that used in [48] in every respect. The governing equations which includes volumetric heating source is solved using the Finite-Element Method with appropriate boundary conditons as reported in used in [48].

The temperature isotherms and corresponding fluid velocity contours are presented in Figure 21a e for angles of inclination, $\phi = 0^\circ, 15^\circ, 30^\circ, 45^\circ$, and 60° at $t = 1250s$, a time within the transitional regime where thermal plumes are evident and convective heat transfer is the highest.

Features at zero inclination have been discussed in the previous section and in Amber and O'Donovan [47]. For inclination angle (Fig 21b) $\phi = 15^\circ$, the effect of this inclination angle on non-uniform temperature profile can be seen. Visual inspection revealed a reduction in the hot stratified layer thickness. Thermal plumes extended further into the fluid column than at $\phi = 0^\circ$ but remained confined by it. At higher inclination angles $\phi = 30^\circ$ (Fig 21c), $\phi = 45^\circ$ (Fig 21d), and $\phi = 60^\circ$ (Fig 21e), natural convection from the lower surface decreased and stable stratification increased with increased inclination angle. Higher Rayleigh promoted natural convection.

Table 2 presents a summary of the numerical investigations on the natural convection induced by the selective absorption of radiation in a fluid layer found in literature.

A systematic comparison of the models developed to date have successfully given insights into the physical processes encountered in induced natural convection in reservoirs. Most of the models have accounted for the depth-dependent absorption of solar radiation within fluid depths using a single value, solar weighted, absorption coefficient and a Total Solar Intensity (TSI) value. The major drawback of applying single bulk values for the absorption coefficient as obtained from an equation is that they are characteristic of the

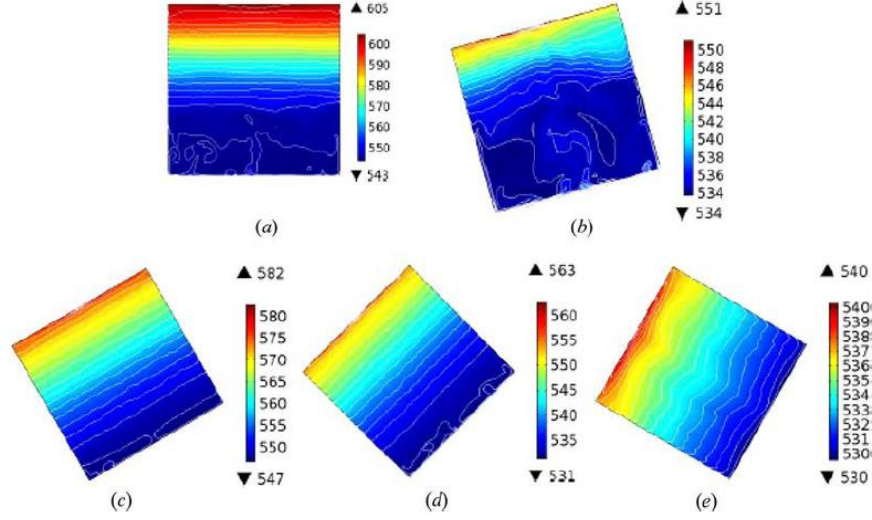


Figure 21: Transient surface temperature plots for ϕ values: (a) 0° (b) 15° (c) 30° (d) 45° (e) 60° [48].

936 visible part of the spectrum, where the salt is mostly transparent to radiation.
 937 As such the models fail to account for the effects of the longer wavelengths
 938 [24]. Other numerical models have been developed which have accounted for
 939 the spectral volumetric absorption of solar radiation in fluid depths, using
 940 spectral band models [52, 49]. In spectral band models, an average extinc-
 941 tion coefficient k_j and an energy component B_j over each wavelength band
 942 are employed to estimate the spectral depth-dependent absorption of solar
 943 radiation in fluid layers. Lenert and Wang [50] used a two-band model in
 944 a numerical study on "Optimisation of Nano fluid volumetric receivers for
 945 solar thermal energy conversion". Webb and Viskanta [51] studied the ab-
 946 sorption of solar radiation and heat transfer in a directly-irradiated, thin,
 947 falling molten salt film based on two- and three-band attenuation models.
 948 Their results showed the sensitivity of their calculations to absorption co-

949 efficient band models where the two-band model was found to give higher
 950 system performance compared with the three band model [51]. Hattori et
 951 al. [52] used a three band attenuation model to study the unsteady nat-
 952 ural convection induced by absorption of solar radiation, with relevance to
 953 littoral waters. Wu et al. [49], studied the volumetric heat release inside
 954 a heat transfer fluid based on a six band attenuation spectral model. The
 955 study made comparisons between results of a six-band spectral model with
 956 those of a twenty-band solar spectrum model and results were said to be in
 957 good agreement [49]. Discrepancies exists between conclusions of the study
 958 of Wu et al. [49] and findings of Webb and Viskanta [51]. The latter reports
 959 considerable differences between a 2 and 3 band model, which are expected
 960 to depend on the resolution of the spectral band. However estimating spec-
 961 tral radiation and wavelength attenuation coefficient of a fluid by bands has
 962 been found to lead to errors. Gueymard [53] discusses the drawbacks of
 963 the band models with regards to solar intensity distribution and shows the
 964 limitations of employing band models over his parametrised models spectra
 965 transmittance models. According to Gueymard [53], using a small number
 966 of wavelengths in the band model limits the accuracy of the modelled atten-
 967 uation, particularly in spectral bands where strong absorption exists.

968

969 5. Conclusion

970 A comprehensive review on the on the up to date progress made on radia-
 971 tion induced natural convection in reservoirs primarily heated by absorption
 972 of solar radiation has been performed and presented. Results from experi-
 973 mental, numerical and theoretical investigations by comparison are seen to
 974 reveal consistent heat transfer and fluid dynamics. Generally, the temper-
 975 ature fields are described as non-linear temperature profile consists of two
 976 distinct layers: stable stratified surface layer and a thermal boundary layer
 977 are present. Flow development reveals three basic flow stages, an initial a
 978 transitional and a quasi steady state stages. The heat transfer mechanisms
 979 and flow structures, and development obtained from the various research ap-
 980 proaches are consistent and in good agreement. The observed challenges and
 981 short comings the respective research approaches for the present problem is
 982 addressed as well.

983

984

Table 2: Summary of the numerical investigations on the natural convection induced by the selective absorption of radiation in a fluid layer in various enclosures.

Author	Geometry	Method	Remarks
Lei and Patterson [6, 7, 9]	Triangular	Two dimensional Finite volume method. SIMPLE scheme adopted for pressure velocity coupling and QUICK scheme applied for spatial derivatives. Second order implicit scheme used for time discretisation. Three dimensional Finite Difference Method. Discretised using second order differencing and upwind scheme. Second order time accurate backward differencing scheme applied to time integration.	Natural convection and flow development from an isothermal and stationary state in a reservoir side arm subject to solar radiation are shown to occur through an initial stage, transitional stage and quasi steady state.
Hattori et al. [29, 35]	Paralleplid	Finite volume SNS code, Adams-Bashforth and CrankNicolson time discretisation schemes, second-order central differencing, ULTRA (universal limiter for tight resolution and accuracy) flux limiter. The strongly implicit procedure (SIP).	Analytical solution for temperature shows temperature stratification consists of an upper stable stratification and a lower unstable stratification. Nonlinear temperature stratification limitation of the mixing driven by rising thermal plumes, stability, properties of the flow are independent of the Prandtl number and
Hill [32]	Rectangular	compound matrix and Chebyshev-tau techniques.	Linear internal heat source from absorption of radiation based on constants of proportionality.
Harfash [33, 34]	cuboid	velocity-vorticity formulation	Linear theory is very accurate in predicting the onset of convective motion, and thus, regions of stability. Convection has three different patterns.
Webb & Viskanta [39]	Rectangular	Discretised using the control volume scheme of Patankar. SIMPLE algorithm was used to couple pressure and momentum.	Volumetric heat source accounted for based on weighted mean absorption coefficient and constant isotropic solar radiation.
Onyegegbu [40]	Rectangular	Finite difference and implicit method. Internal heat generation based on combined mean Planck, Roseland and scattering coefficients	Convection is found to set in as two counter-rotating eddies of unequal strength. Convection is seen to be encouraged by lowering the optical thickness of the fluid and increasing the albedo. The bottom boundary emissivity is important only at low and intermediate optical thicknesses where lowering boundary emissivity suppresses convection.
Verevchkin and Startsev [41]	Rectangular	Finite volume method with expansion of variables in a Fourier series with time dependent coefficients.	Three different regimes found: intermittent convection, steady-state convection, and free convection. The transitions occur at different values of J_0/Q , the ratio of downward solar-radiation flux just below the surface to heat flux through the interface (assumed to be constant), but at almost the same Rayleigh number. The generalized heat-conduction law is found to be valid.
Coates and Patterson [42]	rectangular	Finite Volume method with the SIMPLE scheme, QUICK scheme. Time integration discretised by second order Crank-Nicholson predictor corrector method	Intrusion velocities and transition times determined. Results from code agree with earlier experiments over a range of radiation parameters.

Table 1 (Continued): Summary of the numerical investigations on the natural convection induced by the selective absorption of radiation in a fluid layer in various enclosures.

Author	Geometry	Method	Remarks
Dittko et.al [43]	Tetrahedron domain	Cartesian grid with an Immersed Boundary Method	Significant differences were predicted by the theoretical scaling analysis compared to the triangular domain as presented in the literature. Complex flow patterns were found to be present including helical flow. These features were not accounted for by the theoretical scaling analysis. Despite the complex flow features a number of similar empirical scaling relationships have been found for the tetrahedron cavity based on the numerical results. In particular, the volumetric transfer rate for the cavity was found to vary with the cube root of the Rayleigh number. Relationships were also found for the transition locations between different flow regimes. The original theoretical scaling results were found to provide qualitative information on the locations of different flow regimes and predicted the presence of turbulent flow well. The empirical relationships described have potential applications in large scale lake models. Rather than explicitly resolving sidearms they can be represented by a simple model that uses these relationships as a basis. Further work is needed, however, to better understand the role of the aspect ratios in these models.
Bednarz et.al [44–46]	Triangular	a finite volume method with a double precision solver. A second-order time accurate formulation. Spatial derivatives used a third-order MUSCL scheme conceived by blending a central differencing scheme, second-order upwind scheme, Pressurevelocity and SIMPLE algorithm.	Stable stratification of the water body during the heating phase and an unsteady mixing flow in the reservoir during the cooling phase. Thermal instabilities play an important role in breaking up the residual circulation and initiating a reverse flow circulation in deep waters when the thermal conditions change from heating to cooling. If cooling is sufficiently strong, a clear undercurrent is formed, bringing cold water to the deep region of the reservoir. Heating from the water surface results in a stable large-scale convective roll which is clearly observed in the simulations
Amber & O'Donovan [47, 48]	Square	Finite Element Method with implicit back stepping method. Second-order and linear elements discretisation were applied for the velocity and the pressure field, (P_2+P_1) .	Nonlinear temperature profile consisting of distinct layers where thermocapilarity and buoyancy effects are evident. Fluid flow development in the lower layer is found to vary significantly with time and exhibits an initial stage, transitional stage and quasi-steady stages. The magnitude of the natural convection and the duration of each stage is found to decrease as the aspect ratio increases from 0.5 to 2. Calculation of the average heat transfer reveals that the Nusselt Rayleigh number relationship is not uniformly linear and the average heat transfer over the lower boundary surface increased with increasing Ra. Increasing the inclination angles decreases the natural convection and increases stratification. Higher Rayleigh number promotes natural convection.

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